Indian Statistical Institute, Bangalore

B. Math.(Hons.) II Year, First Semester

Mid-Sem Examination

Analysis -III

Time: 3 hours September 10, 2009 Instructor: B.Rajeev

Maximum Marks 40

1. Let $\Omega \subseteq \mathbb{R}^n$ be open and connected. Let $f : \Omega \longrightarrow \mathbb{R}^n$ be a continuous vector field. Then show that $f = \bigtriangledown \phi$ for some continuously differentiable $\phi : \Omega \longrightarrow \mathbb{R}$ if for every $x, y \,\varepsilon\, \Omega$ and every piecewise smooth curve $r : [a, b] \longrightarrow \Omega$ with $r(a) = x,\ r(b) = y$ the line integral $\int f.dr$ is independent of r. [3+7]

2. Verify gauss divergence theorem for a tetrahedron with vertices at $0, A, B, C,$ where $0 = (0, 0, 0,)\quad A - (a, 0, 0), B = (0, b, 0)$ and $C = (0, 0, c)$. Here you can assume $a, b, c > 0$. [8]

3. Let $f(t) = t + 2t^2 sin(\frac{1}{t})$. Show that f is not $1 - 1$ on any neighborhood of 0. [4]

4. Let $(X, d), (Y, m)$ be metric spaces and (Y, m)be complete. Let $(C_b(X, Y), E)$ be the metric space given by

   $E(f, g) = \sup\limits_{x \varepsilon X} m[f(x), g(x)]$. Then show that $(C_b(X, Y), E)$ is a complete metric space. [8]

5. Let $f[x, y) = \frac{x^2 - y^2}{(x^2+y^2)^2}$ for $0 \leq x, y \leq$. Show that iterated internals exist for f and are not equal. [5]

6. Let $\shortmid_q$ = all polynomials in a single variable x with rational co effects. Show that $\shortmid_q$ is a dense subset of $(C^1[a, b], D)$ Where $D(f, g) =$ sup $\{|f(x0 - g(x)|f^1(x) - g^1(x)|\}a \leq x \leq b$. [5]

7. Let $g : (a, b] \longrightarrow R$ be a bounded continuous function. if $a_n \longrightarrow a$, then show that $\int_{a_n}^{b} g|t|$ all is convergent as $n \longrightarrow \infty$ and the limit is independent of the sequence $a_1, a_2, \cdots$ [3]

8. Let $f_n(t) = n^2 t(1 - t)^n$ for $0 \leq \in \leq 1$. Show that $f_n(t) \longrightarrow 0$ as $n \longrightarrow \infty$ and $\int\limits_0^1 f_n(t)$ all $\longrightarrow R$ as $n \longrightarrow \infty$. [2]

9. Let$\partial : [a, b] \longrightarrow R$ be any increasing function. Let $f_n : [a, b] \longrightarrow R$ be all bounded and Riemann - stieltjets integrable $W, r, t\partial$. Let f :

$[a, b] \longrightarrow R$ be a bounded function such that lim sup $|f_n(t) - f(t)| =$
$n \longrightarrow \partial a \leq t \leq b$

Then

(a) $\int_a^b f d\partial$ exists and

(b) $\int_a^b f_n d\partial \longrightarrow \int_a^b f d\partial$ [6]